Distributed Optimization for Machine Learning

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Lecture 2 – Convex Optimization

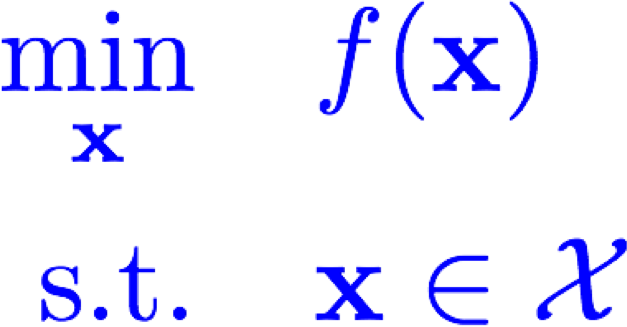
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# Recap

* Global and local optimality for unconstrained optimization
* Necessary and sufficient optimality conditions
* Difference between convex and non-convex optimization
* Convexity + optimality condition è Global optimality

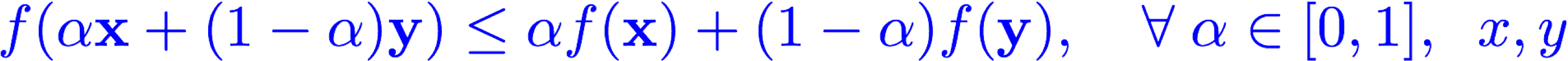
Convex Optimization Problem?

**Convex function Convex Set**



* Is a given function convex?
* Is a given set convex?

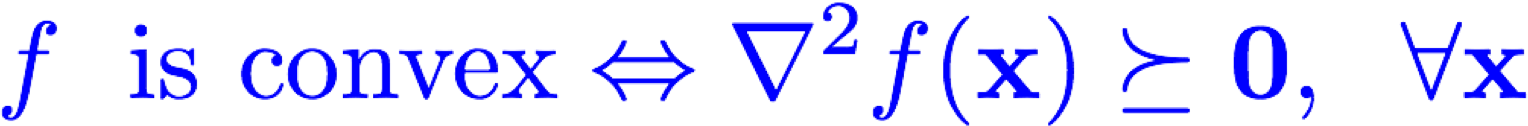
How to identify a convex function?

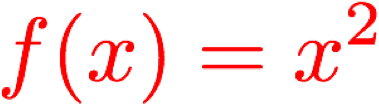
* Definition: 

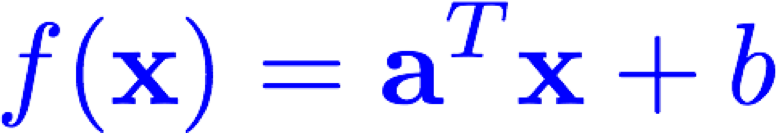
**(Very useful definition: Jensen’s inequality)**

* Hessian for twice differentiable functions:

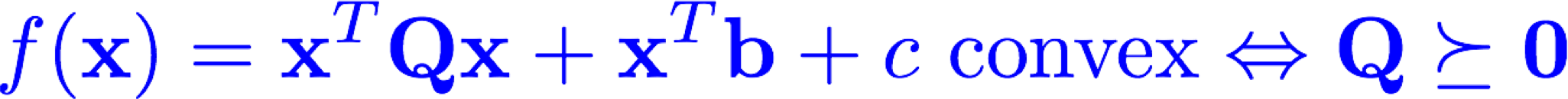
**Examples:**

* Affine functions:



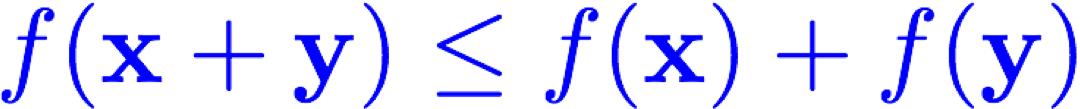
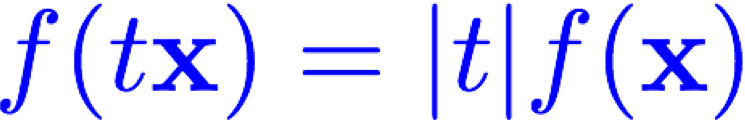
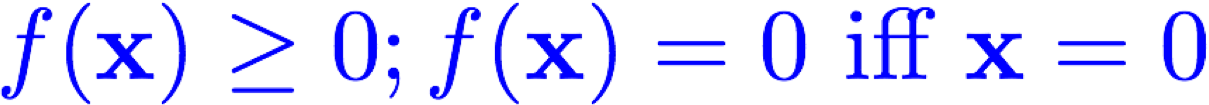


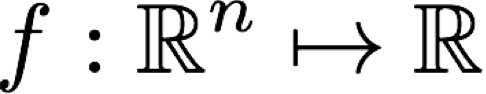
* Quadratic functions:

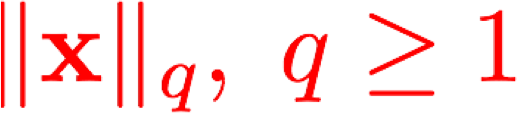


Convex Functions

* Norms:**Examples:**



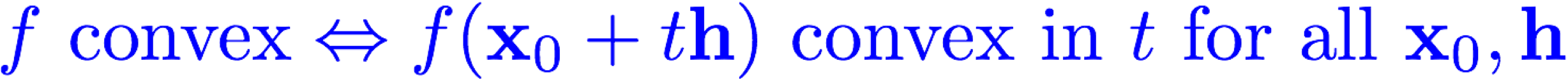
is a norm if

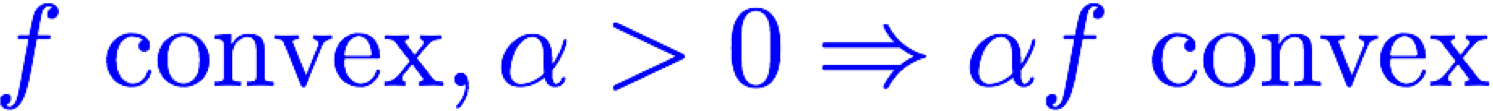
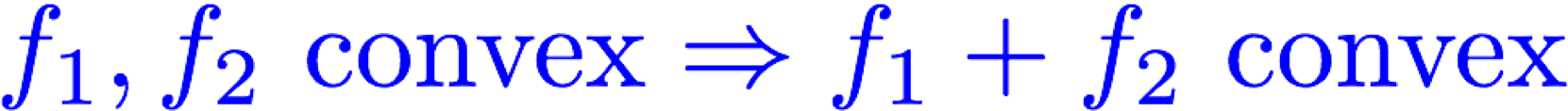


Special cases



* A function is convex iff it is convex on every line:

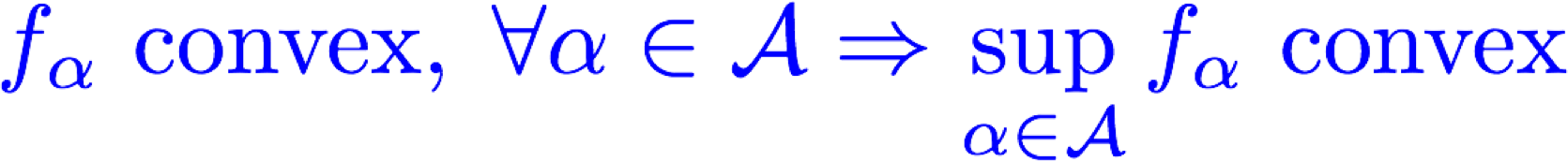


* Positive multiple of convex function 
* Sum of convex functions:  Extension to integral

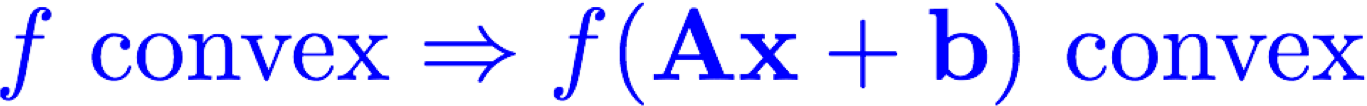
# Convex Functions

* Pointwise supremum of convex functions

Example

 **No assumption on** 

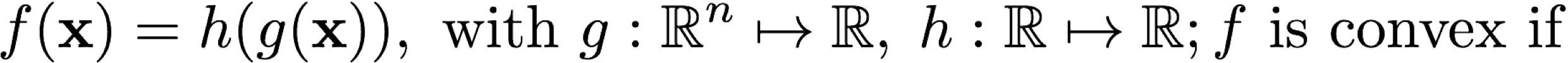
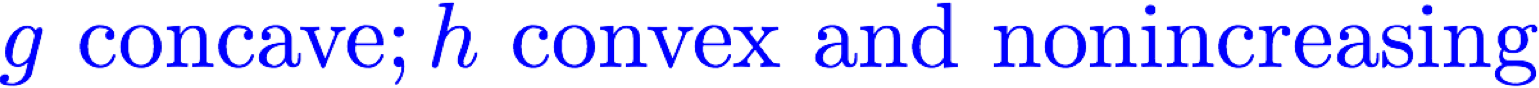
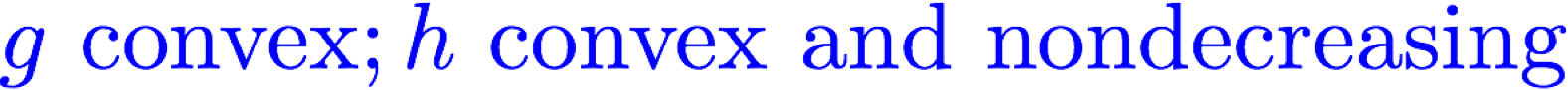
* Affine transformation of domain



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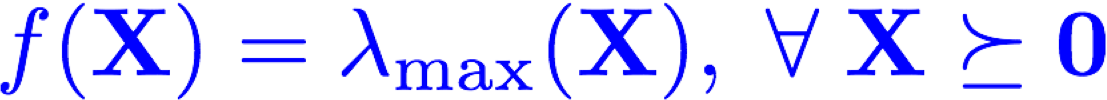
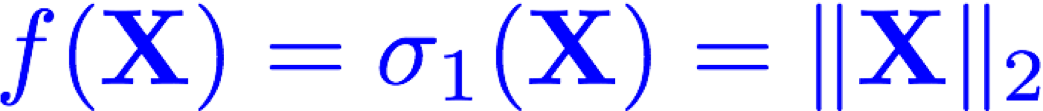
Composition:



Necessary?

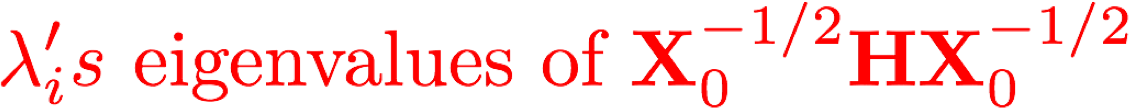
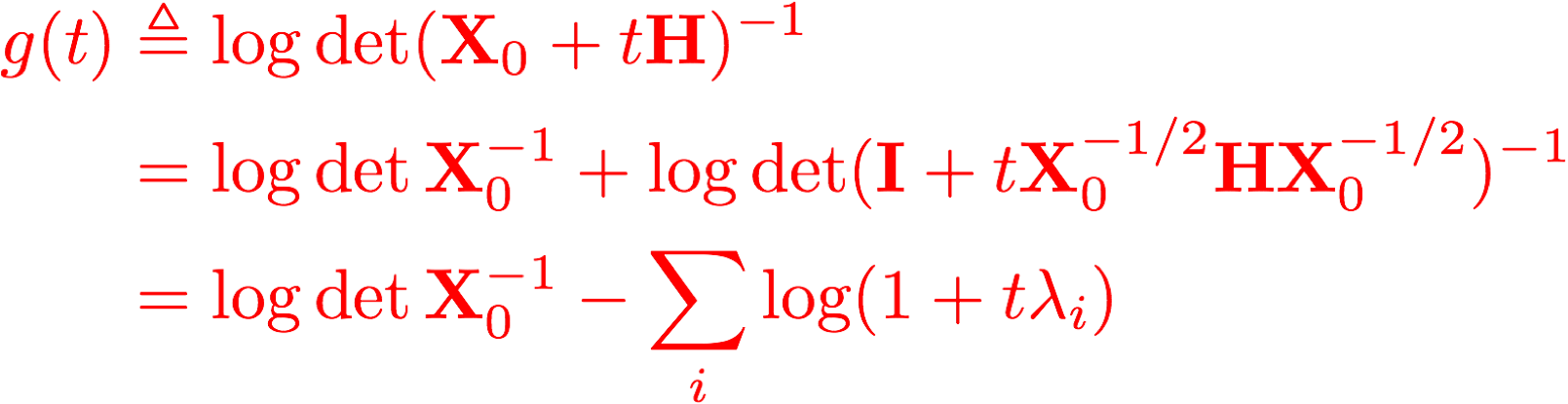
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Examples of Convex Functions on Matrices

* Affine function: 
* Largest eigenvalue on symmetric matrices: 
* Largest singular value: 

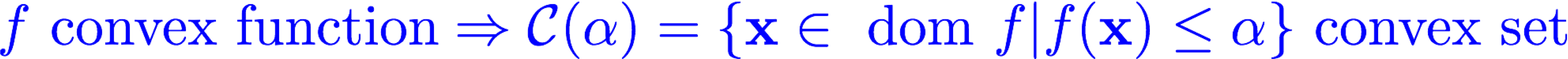


* Log-det-inv on PSD matrices 



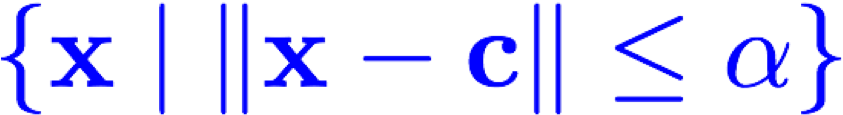
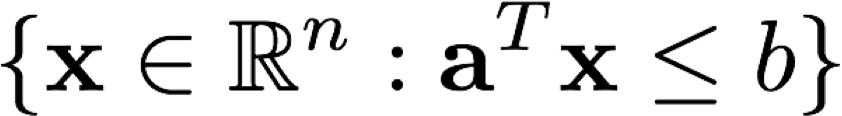
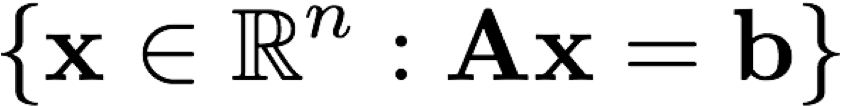
# From Convex Functions to Convex Sets

* A sublevel set of a convex function is convex set



**Is the converse true?**

Examples of Convex Sets

* Affine sets: 

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Set of PSD matrices

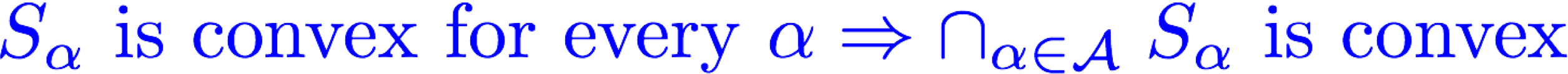
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Halfspace

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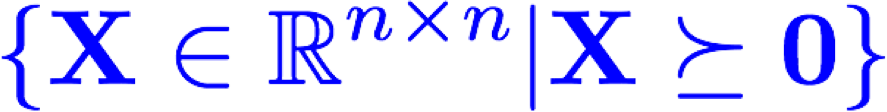
Intersection of convex sets



**True for union?**

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Norm balls



**Change it to infinite?**

**Example:**

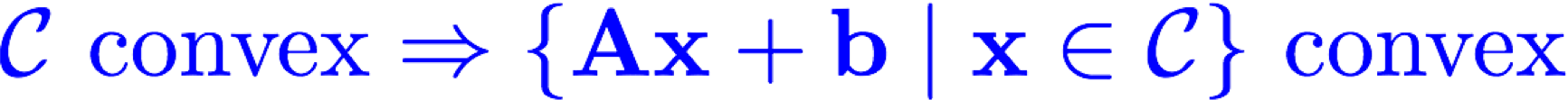
**Polyhedron: intersection of finite number of**

**halfspaces**

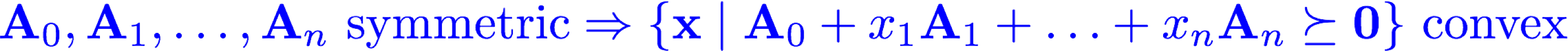
# Convex Sets

* Affine transformation of convex sets:





* Linear matrix inequalities



* A popular way of representing some convex optimization problems:

